Published Online August 2012

www.iji-cs.org



# A Fuzzy-Neural-Monte Carlo Method to Estimate Online Content Delivery Cost in E-learning

Hamed Fazlollahtabar<sup>1</sup>, Tabande Sherafat <sup>2</sup>, Alireza Es'haghzadeh <sup>3</sup> <sup>1</sup>Faculty of Industrial Engineering, Iran University of Science and Technology, Tehran, Iran <sup>2</sup>Department of Information Technology Engineering, Mazandaran University of Science and Technology, Babol, Iran <sup>3</sup>Department of Industrial Engineering, Mazandaran University of Science and Technology, Babol, Iran \*E-mail: hfazl@iust.ac.ir

(Abstract) This paper concerns with an economic analysis in e-learning environment. Regarding to the virtual nature of e-learning, uncertainties are inevitable. Learning consists of different integrated factors and due to real-time structures of e-learning in online content delivery, the components cost analysis help the decision makers to analyze the system economically. Here, the aim is proposing a comprehensive economic analysis model considering uncertainties in online course delivery. The input data being the online content delivery factors are considered to be fuzzy and the processing unit is a neural network based on Monte Carlo simulation as an adjustment core. The output of the proposed neural network is the cost estimation. The effectiveness and applicability of the proposed methodology is illustrated in a numerical example.

Keywords: E-Learning; Online Content Delivery; Cost Estimation; Fuzzy Sets; Neural Network; Monte Carlo Simulation

#### 1. INTRODUCTION

Despite the extensive development of the information technologies, information alone is not decisive to assure a competitive advantage. Global communication networks like the Internet enable the almost immediate exchange of information without limits of place and time [1]. However, it is not the information alone, but the knowledge and capability to assess, judge and apply these information in the process of a learning process [2]. Distance education is widely recognized as the alternative delivery system in which the student and the educator are separated either by distance or time or, in some cases, both. However, distance education and virtual learning are not new concepts, but rather have evolved from the ubiquitous correspondence courses of the past [3]. In the modern implementation, information or distributed learning technology is the likely connector between the learner, the instructor, and the offering site. Education at a distance may be as near as the on-campus residence halls or as far as a distant workplace [4].

Virtual learning is a term used to describe learning activities that can be accessed in a setting that is free from time and location constraints. Virtual learning comes in many forms, but it is about using computers to access information, courses and tools to develop skills and knowledge on all types of subjects. Virtual learning provides convenient and just-in-time access to learning programs and professional development materials. Virtual learning is learning that is enabled or supported by the use of digital

tools and content [1]. It typically involves some form of interactivity, which may include online interaction between the learner and their teachers or peers. Virtual learning opportunities are usually accessed via the internet, though other technologies such as CD-ROM are also used in virtual learning. The last few years have seen a wave of digital tools and content being designed to facilitate the learning process [5]. Virtual learning enables different types of learning activities from those that rely on traditional teaching modes (e.g. the face-to-face lecture, tutorial or lab), and traditional media (e.g. books). It is increasingly used to support campus-based courses, and is bringing new dimensions to distance education [6].

Here, we propose a model to evaluate economic aspects of e-learning considering the uncertain parameters in online content delivery. Any e-learning is configured integrating a set of factors for online interaction. To evaluate the e-learning from cost view point, the factors should be assessed. Due to uncertainty in the status of the factors, an integrated system is designed to control the uncertainty and do the cost estimation computations.

#### 2. LITERATURE REVIEW

For comparing the e-learning system and traditional learning system, two comprehensive studies were illustrated. Mahdavi et al. [7] compared traditional system with virtual educational system statistically in Iran. By the means of economical equations and statistical analysis they illustrated an in depth survey. They illustrated the best option for educational system is the combination of both systems. Fazlollahtabar and Sharma [8] compared traditional engineering educational system with the e-learning engineering educational system on the economic dimension using hypothesis testing approach in Iran. The comparison involved trend analysis and prediction based on costs and benefits of the two systems. Interestingly, the analysis revealed that the traditional system had greater advantage on the economic dimension. Several factors support the e-learning system despite the associated economic disadvantage. The final analysis provided results in favor of a blended system which takes advantage of the traditional and e-learning systems.

Different studies have been worked out on cost optimization within e-learning environment. Mahdavi et al. [9] identified varied cost elements in e-learning educational system and optimized them by the means of mathematical programming. Then they proposed an effective method to estimate the learning cost between any two skills of learner using the grey relational analysis. Mahdavi et al. [10] developed their study combining the grey relational analysis and a radial basis function network to estimate the learning cost between any two skills after identification of varied cost elements in e-learning educational system and optimization by the means of mathematical programming. Fazlollahtabar and Yousefpoor [11] applied the cost elements in the e-learning educational systems and proposed a combination of grey relational analysis and a radial basis function network to estimate the learning cost between any two skills. An integer programming method was employed to demonstrate that it is possible to facilitate the acquisition of single skills by considering a set of useful compound skills.

Fazlollahtabar and Mahdavi [12] proposed a neuro-fuzzy approach based on an evolutionary technique to obtain an optimal learning path for both instructor and learner. The neuro-fuzzy implementation helps to encode both structured and non-structured knowledge for the instructor. On the other hand, for learners, the neural network approach has been applied to make personalized curriculum profile based on individual learner requirements in a fuzzy environment.

## 3. PROBLEM STATEMENT

We consider an e-learning system for online content delivery to users. The components of such a system are hardware, software and human resource. Due to real-time structure of the system controlling decisions are accompanied with uncertainty. The major controlling decision in e-learning is economic view, cost estimation. Therefore, the costs and benefits of the system are treated using uncertain data being considered to be fuzzy. Since the estimation is performed using some sub-factors for hardware, software and human resource, a common and reliable method to relate sub-factors with main factors is neural network (NN). A configuration of

neural network is shown in Figure 1.

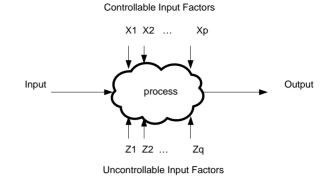


Figure 1. A configuration of NN

Also, we program the model for different time periods for the dynamism associated with the e-learning system. Thus, several outputs are obtained. To get unique cost estimation for the main components a Monte Carlo simulation is employed. The graphical presentation of the proposed methodology is shown in Figure 2.

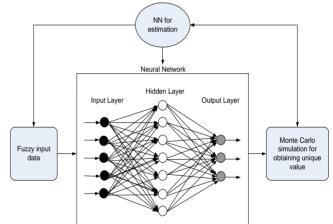


Figure 2. The proposed methodology

The basic materials are given next.

# 4. BASIC MATERIALS

## 4.1. Fuzzy Sets

Here to imply the uncertainties in the VLEs, some basic definitions of fuzzy sets and fuzzy numbers are reviewed from [13, 14]. The basic definitions and notations will be used throughout the paper.

**Definition 1.** A fuzzy set  $\tilde{A}$  in a universe of discourse X is characterized by a membership function  $\mu_{\tilde{A}}(x)$  which associate ©es with each element x in X a real number in the interval [0, 1]. The function value  $\mu_{\tilde{A}}(x)$  is termed the grade of membership of x in  $\tilde{A}$  (Buckley, 1987).

**Definition 2**. A fuzzy number  $\tilde{A}$  is a fuzzy convex subset of the real line satisfying the following conditions:

(a)  $\mu_{\tilde{A}}(x)$  is piecewise continuous; (b)  $\mu_{\tilde{A}}(x)$  is normalized, that is, there exists  $m \in \Re$  with  $\mu_{\tilde{A}}(m) = 1$ , where m is called the mean value of  $\tilde{A}$  [15].

**Definition 3.** A triangular fuzzy number  $\tilde{a}$  can be defined by a triplet  $(a_1, a_2, a_3)$ . Its conceptual schema mathematical form are shown by Equation (1):

$$\mu_{\bar{a}}(x) = \begin{cases} 0, & x \le a_1 \\ \frac{x - a_1}{a_2 - a_1}, & a_1 < x \le a_2 \\ \frac{a_3 - x}{a_3 - a_2}, & a_2 < x \le a_3 \\ 0, & a_3 < x. \end{cases}$$

$$(1)$$

A triangular fuzzy number  $\tilde{a}$  in the universe of discourse X that conforms to this definition has been shown in Figure 3.

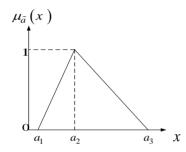


Figure 3. A triangular fuzzy number  $\tilde{a}$ .

**Definition** 4. Assuming that both  $\tilde{a} = (a_1, a_2, a_3)$  and

 $b = (b_1, b_2, b_3)$  are triangular numbers, then the basic fuzzy operations are:

$$\tilde{a} \times \tilde{b} = (a_1 \times b_1, a_2 \times b_2, a_3 \times b_3)$$
 for multiplication (2)

$$\tilde{a} + \tilde{b} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$$
 for addition (3)

**Definition 5.** The  $D_{p,q}$  -distance, indexed by parameters

1 and <math>0 < q < 1, between two fuzzy numbers

 $\tilde{a}$  and  $\tilde{b}$  is a nonnegative function given by:

$$D_{p,q}(\tilde{a},\tilde{b}) = \begin{cases} \left[ (1-q) \int_0^1 \left| a_\alpha^- - b_\alpha^- \right|^p d\alpha + q \int_0^1 \left| a_\alpha^+ - b_\alpha^+ \right|^p d\alpha \right]^{\frac{1}{p}}, & p < \infty, \ (4) \\ (1-q) \sup_{0 < \alpha < 1} \left( \left| a_\alpha^- - b_\alpha^- \right| \right) + q \inf_{0 < \alpha < 1} \left( \left| a_\alpha^+ - b_\alpha^+ \right| \right), & p = \infty. \end{cases}$$

# 4.2. The Backpropagation Neural Network

The backpropagation algorithm trains a given feed-forward multilayer neural network for a given set of input patterns with known classifications. When each entry of the sample set is presented to the network, the network examines its output response to the sample input pattern. The output response is then compared to the known and desired output and the error value is calculated. Based on the error, the connection weights are adjusted. The backpropagation algorithm is based on Widrow-Hoff delta learning rule in which the weight adjustment is done through mean square error of the output response to the sample input.

## Algorithm 1: General steps of backpropagation.

- 1. Propagate inputs forward in the usual way, i.e., all outputs are computed using sigmoid thresholding of the inner product of the corresponding weight and input vectors. All outputs at stage n are connected to all the inputs at stage n+1
- 2. Propagate the errors backwards by apportioning them to each unit according to the amount of the error the unit is responsible for.

We now discuss how to develop the stochastic backpropagation algorithm for the general case. The derivation is simple, but unfortunately the book-keeping is a little messy. The following notations and definitions are needed:

 $\vec{x}_{i}$ : input vector for unit j ( $x_{ji} = i$ th input to the jth unit)

 $\vec{w}_{j}$ : weight vector for unit j ( $w_{ji}$  = weight on  $x_{ji}$ )

 $z_i = \vec{w}_i \cdot \vec{x}_i$ : the weighted sum of inputs for unit j

 $o_j$ : output of unit  $j(o_i = \sigma(z_i))$ 

 $t_i$ : target for unit j

Downstream(i): set of units whose immediate inputs include the output of i

Output: Set of output units in the final layer.

Since we update after each training example, we can simplify the notation somewhat by assuming that the training set consists of exactly one example and so the error can simply be denoted by E.

We want to calculate  $\frac{\partial E}{\partial w_{ii}}$  corresponding to each input

weight  $w_{ii}$  of each output unit j. Note first that since  $z_i$  is a function of  $w_{ii}$  regardless of where in the network unit j is located,

$$\frac{\partial E}{\partial w_{ji}} = \frac{\partial E}{\partial z_j} \cdot \frac{\partial z_j}{\partial w_{ji}} = \frac{\partial E}{\partial z_j} \cdot x_{ji}, \tag{5}$$

Furthermore,  $\frac{\partial E}{\partial z_i}$  is the same regardless of which input

weight of unit i we are trying to update. So, we denote this

quantity by  $\delta_i$ .

Consider the case when i is an output unit. We know that

$$E = \frac{1}{2} \sum_{k \in \text{Outputs}} (t_k - \sigma(z_k))^2$$
 (6)

Since the outputs of all units  $k \neq j$  are independent of  $w_{ji}$ , we can then drop the summation and consider just the contribution to E by j and we call it  $\delta_j$ :

$$\delta_{j} = \frac{\partial E}{\partial z_{j}} = \frac{\partial}{\partial z_{j}} \frac{1}{2} (t_{j} - o_{j})^{2} = -(t_{j} - o_{j}) \frac{\partial o_{j}}{\partial z_{j}} = -(t_{j} - o_{j}) \frac{\partial}{\partial z_{j}} \sigma(z_{j})$$
(7)  
$$= -(t_{j} - o_{j}) (1 - \sigma(z_{j})) \sigma(z_{j}) = -(t_{j} - o_{j}) (1 - o_{j}) o_{j}.$$

Thus.

$$\Delta w_{ji} = -\eta \frac{\partial E}{\partial w_{ji}} = \eta \delta_j x_{ji}. \tag{8}$$

Now, consider the case when j is a hidden unit. Like before, we make the following two important observations:

- 1. For each unit k downstream from j,  $z_k$  is a function of  $z_i$ .
- 2. The contribution to error by all units  $l \neq j$ , in the same layer as j, is independent of  $w_{ii}$ .

We want to calculate  $\frac{\partial E}{\partial w_{ji}}$  for each input weight  $w_{ji}$  for each

hidden unit j. Note that  $w_{ji}$  influences just  $z_j$  which influences  $o_j$  which influences  $z_k$ ,  $\forall k \in Downstream(j)$ , each of which influences E. So, we can write,

$$\frac{\partial E}{\partial w_{ji}} = \sum_{k \in Downstream(j)} \frac{\partial E}{\partial z_k} \cdot \frac{\partial z_k}{\partial z_k} \cdot \frac{\partial o_j}{\partial z_j} \cdot \frac{\partial z_j}{\partial w_{ji}} = \sum_{k \in Downstream(j)} \frac{\partial E}{\partial z_k} \cdot \frac{\partial z_k}{\partial o_j} \cdot \frac{\partial o_j}{\partial z_j} \cdot x_{ji} \cdot {}^{(9)}$$

Again, note that all the terms except  $x_{ji}$  in (9) are the same regardless of which input weight of unit j we are trying to update. Like before, we denote this common quantity by  $\delta_j$ .

Also, note that 
$$\frac{\partial E}{\partial z_k} \delta_k$$
,  $\frac{\partial z_k}{\partial o_j} w_{kj}$  and  $\frac{\partial o_j}{\partial z_j} = o_j (1 - o_j)$ .

Substituting them in (7),

$$\delta_{j} = \sum_{k \in Downstream(j)} \frac{\partial E}{\partial z_{k}} \cdot \frac{\partial z_{k}}{\partial o_{j}} \cdot \frac{\partial o_{j}}{\partial z_{j}} = \sum_{k \in Downstream(j)} \delta_{k} \cdot w_{kj} \cdot o_{j} (1 - o_{j}), \quad (10)$$

we obtain:

$$\delta_k = o_j (1 - o_j) \sum_{k \in Downstream(j)} \delta_k . w_{kj} . \tag{11}$$

# 4.3. Fuzzy neural networks

Fuzzy neural networks (FNN) have been proposed as a knowledge engineering technique and used for various application domains by several authors including Yamakawa et al. [16]. Fuzzy sets were initially introduced into neural networks in 1993 [17], but has not experienced much development until recently. Since 1995, much attention has been focused on incorporating fuzzy reasoning into neural networks [18]. A neural network is considered a fuzzy neural network if the signals and/or the weights in the system are based around fuzzy sets [19]. The fuzzy neural network systems (Neuro-fuzzy systems) combine the advantages of fuzzy logic systems and neural networks have become a very active subject in many scientific and engineering areas, such as, model reference control problems, PID controller tuning, signal processing, etc. The fuzzy neural network (FNN) system is one kind of fuzzy inference system in neural network structure [20, 21, 22, 23].

#### 4.4. Monte Carlo simulation

Monte Carlo simulation is a comprehensive approach for analyzing the behavior of some activities, plans or processes that involve uncertainty. If we face uncertain or variable market demand, fluctuating costs, variation in a manufacturing process, or effects of weather on operations, or stochastic activity time we can benefit from using Monte Carlo simulation to understand the impact of uncertainty, and develop plans to mitigate or otherwise cope with risk. Whenever we need to make an estimate, forecast or decision where there is significant uncertainty, we'd be well advised to consider Monte Carlo simulation [24].

Monte Carlo simulation is a method for iteratively evaluating a deterministic model using sets of random numbers as inputs. This method is often used when the model is complex, nonlinear, or involves more than just a couple uncertain parameters. The Monte Carlo method is just one of many methods for analyzing uncertainty propagation, where the goal is to determine how random variation, lack of knowledge, or error affects the sensitivity, performance, or reliability of the system that is being modeled. Monte Carlo simulation is categorized as a sampling method because the inputs are randomly generated from probability distributions to simulate the process of sampling from an actual population. So, we try to choose a distribution for the inputs that most closely matches data we already have, or best represents our current state of knowledge. The data generated from the simulation can be represented as probability distributions (or histograms) or converted to error bars, reliability predictions, tolerance zones, and confidence intervals.

The following five steps are proposed to implement Monte

Carlo,

**Step 1:** Create a parametric model,  $y = f(x_1, x_2, ..., x_q)$ .

**Step 2:** Generate a set of random inputs,  $x_{i1}$ ,  $x_{i2}$ , ...,  $x_{iq}$ .

**Step 3:** Evaluate the model and store the results as  $y_i$ .

**Step 4:** Repeat steps 2 and 3 for i = 1 to n.

**Step 5:** Analyze the results using histograms, summary statistics, confidence intervals, etc.

#### 5. PROBLEM FORMULATION

Here, we formulate our proposed model including uncertainty in economic evaluation of e-learning online content delivery factors. Initially, we determine the effective factors in online content delivery to be hardware, software and human resource. Learning in e-learning is an integrated network of some factors. Learning depends on the suitable functioning of hardware (servers, computers, ...), software (learning management systems), and human (instructor, administrator, ...). Each of these factors could be assessed economically. While the status of these factors associated with uncertainty, we need a method to evaluate them during time. To identify whether a factor is in its appropriate status or not, two economic expressions are applied. Net present value and benefit to cost ratio are two economic expressions that are used to evaluate the economic status of a factor.

The economic model is as follows:

r: rate of return

t: unit of time

 $B_t$ : benefit at time t

 $C_t$ : cost at time t

NPV: net present value

BCR: benefit to cost ratio

To gain the net present value we use.

$$NPV = \sum_{t=1}^{t=T} \frac{\left(B_t - C_t\right)}{\left(1 + r\right)^t}$$
 (12)

In fuzzy case we have,

$$NP\widetilde{V} = \sum_{t=1}^{t=T} \frac{\left(\widetilde{B}_t - \widetilde{C}_t\right)}{\left(1 + \widetilde{r}\right)^t}$$
(13)

To obtain benefit to cost ratio we use.

$$BCR = \frac{\sum_{t=1}^{t=T} \frac{(B_t)}{(1+r)^t}}{\sum_{t=1}^{t=T} \frac{(C_t)}{(1+r)^t}}$$
(14)

In fuzzy case we have,

$$BC\widetilde{R} = \frac{\sum_{t=1}^{t=T} \frac{\left(\widetilde{B}_{t}\right)}{\left(1+\widetilde{r}\right)^{t}}}{\sum_{t=1}^{t=T} \frac{\left(\widetilde{C}_{t}\right)}{\left(1+\widetilde{r}\right)^{t}}}$$
(15)

These equations are used to estimate the cost and benefit in a neural network proposed in Section 4.2. To do the computations in fuzzy environment for all of the factors we use equations (13) and (15). Consider that all costs, benefits, and rate of returns are uncertain and triangular fuzzy numbers. As we described in Section 4.1, we do fuzzy basic operations to determine  $NP\tilde{V}$  and  $BC\tilde{R}$  for each factor. After that, we have to compare the resulted  $NP\tilde{V}$  and  $BC\tilde{R}$  to gain the optimal (maximum) one. This means that it is necessary to have a method for ranking or comparing fuzzy numbers. An operator  $\preceq$  for ordering fuzzy numbers can be defined as follows:

$$\tilde{A} \leq \tilde{B} \iff (a_1 \leq b_1) \wedge (a_2 \leq b_2) \wedge (a_3 \leq b_3) \wedge (a_4 \leq b_4). \tag{16}$$

However, this relation is not a complete ordering, as fuzzy

numbers  $\tilde{A}$ ,  $\tilde{B}$  satisfying

$$\exists i, j \in \{1, 2, 3, 4\}) : (a_i < b_i) \land (a_i > b_i)$$
(17)

are not comparable by  $\prec$ .

Here, we introduce a new fuzzy ranking method for fuzzy numbers. Let us consider fuzzy *max* operation defined in the following way:

Max value
$$(\tilde{a}, \tilde{b}) = (\max(a_1, b_1), \max(a_2, b_2), \max(a_3, b_3), \max(a_4, b_4)).$$
 (18)

It is evident that, for non-comparable fuzzy numbers  $\tilde{a}, \tilde{b}$ , the fuzzy max operation results in a fuzzy number different from both of them. To alleviate this drawback, we propose a new method based on the distance between fuzzy numbers. We use the distance function introduced in [25]. The analytical properties of  $D_{p,q}$  depend on the first parameter p, while the second parameter q of  $D_{p,q}$  characterizes the subjective weight attributed to the end points of the support; i.e.,  $(a_{\alpha}^+, a_{\alpha}^-)$  of the fuzzy numbers. If there is no reason for distinguishing

any side of the fuzzy numbers,  $D_{p,\frac{1}{2}}$  is recommended.

Having q close to 1 results in considering the right side of the support of the fuzzy numbers more favorably. Since the significance of the end points of the support of the fuzzy numbers is assumed to be the same, then we consider  $q=\frac{1}{2}$ . For triangular fuzzy numbers

$$\tilde{a}=\left(a_1,\,a_2,\,a_3\right)$$
 and  $\tilde{b}=\left(b_1,\,b_2,\,b_3\right)$ , the above distance with  $p=2$  and  $q=\frac{1}{2}$  is then calculated as:

$$D_{2,\frac{1}{2}}(\tilde{a},\tilde{b}) = \sqrt{\frac{1}{6} \left[ \sum_{i=1}^{3} (b_i - a_i)^2 + (b_2 - a_2)^2 + \sum_{i \in [1,2]} (b_i - a_i)(b_{i+1} - a_{i+1}) \right]}.$$
 (19)

Thus, we can acquire the distance between  $\tilde{a}$ ,  $\tilde{b}$  and  $M_a \tilde{V}$  using the proposed distance function. We proposed the following algorithm for the comparison of fuzzy numbers. Algorithm 2: Comparing two triangular fuzzy numbers

**Input:** Two fuzzy numbers ( $\tilde{L}_i$ , i = 1, 2)

 $extit{Output:}$  Maximum between two triangular fuzzy numbers  $(\widetilde{L}^{ ext{max}})$ 

Step 1: Calculate the maximum value (  $M_a \widetilde{V}$  ) by Equation (8); (  $M_a \widetilde{V}$  stands for Max Value).

Step 2: Find the distance of  $M_a \tilde{V}$  from  $(\tilde{L}_i, i=1,2)$  using Equation (19).

**Step 3:** Determine  $\widetilde{L}^{\max}$  with the highest distance.

Therefore, we gain several estimations for component and using Monte Carlo simulation a unique value is in hand.

# 6. NUMERICAL EXAMPLE

To illustrate the effectiveness and applicability of the proposed methodology a short numerical example is illustrated. Input data are collected from the experts for the components and inserted to the NN for cost estimation purpose. A feedforward backpropagation network can be used to approximate a function to relate the sub-factors to the main factors. To facilitate the computations of backpropagation neural network, MATLAB 7 user interface, NNtool, was applied. A feedforward network was programmed with sub-factor inputs, ten hidden units with logistics activation function, and the outputs. A configuration of the feedforward neural network is shown in Figure 4.

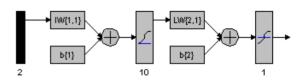


Figure 4. A configuration of the feedforward neural network

Using NNtool, the data were inserted and the required settings were made to train the data to obtain an appropriate pattern. An applied user interface is presented in Figure 5.

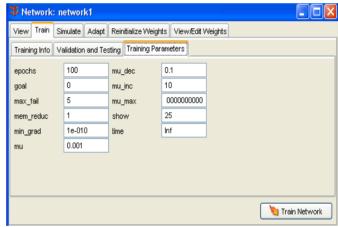


Figure 5. A user interface in MATLAB

Note that, the input data were triangular fuzzy numbers to imply the uncertainty of the data. Now, we apply Monte Carlo simulation to obtain the unique value for each main component as explained before. According to a sampling procedure of occurrences, we compute the variable cost corresponding the cumulative probability of occurrence for each component which is reported in Table 1.

Hardware Software Human resource Cumulative Probability **Cumulative Probability** Sorted times Sorted times Cumulative Probability Sorted times 0.20 0.08 3.8 1.65 0.15 9.6 0.45 3.9 0.026 0.28 9.7 1.85 0.75 4 0.48 2 0.53 9.9 2.1 0.85 4.1 0.72 0.73 10 0.93 4.2 0.9 10.3 0.88 2.2 1 4.25 1 2.25 1 10.35

Table 1. The variable times corresponding the probability of occurrence

Here, according to the Monte Carlo process we generate five series of random numbers as given in Table 2.

Table 2. The generated random numbers

	HARDWARE	SOFTWARE	HUMAN RESOURCE
1	0.632175	0.642	0.381758
2	0.238231	0.0211	0.446354
3	0.567427	0.1123	0.278362
4	0.613292	.78532	0.872533
5	0.013479	0.34578	0.253833

Now, we accommodate the random numbers of each component with the corresponding component estimation in Table 1. The following component cost estimation, Table 3, is obtained for the components associated with the average cost for each component of e-learning.

Table 3. The obtained activity times

	HARDWARE	SOFTWARE	HUMAN RESOURCE
1	4	10	2
2	3.9	9.6	2
3	4	9.6	2
4	4	10.3	2.2
5	3.8	9.9	1.85
Average	3.94	9.88	2.01

Considering the aggregation rule, we have the following cost estimation:

Total estimated cost= 3.94+9.88+2.01=15.83,

Then, the total estimated cost equals to 15.83 unit of money.

# 7. CONCLUSIONS

Here, we proposed an integrated methodology based on fuzzy, neural network and Monte Carlo simulation to analyze the economic status of uncertain factors of e-learning. For economic assessment net present value and benefit to cost ratio were applied. All factors were considered triangular fuzzy numbers and using economic equations and fuzzy basic operations the neural network performed the cost estimation and fuzzy values as the output were obtained in different time periods. To compare those fuzzy numbers an algorithm was introduced and the ranking of e-learning factors is gained. The Monte Carlo simulation was employed to aggregate the time based dynamic values and obtained unique cost estimation for each component for online content delivery in e-learning. The applicability of the methodology was illustrated in a numerical study.

## **REFERENCES**

- [1] Lau, A. and Tsui, E., (2009). Knowledge management perspective on e-learning effectiveness, Knowledge-Based Systems, 22 (4), 324-325.
- [2] Chu, H.C., Chen, T.Y., Lin, C.J., Liao, M.J. and Chen, Y.M., (2009). Development of an adaptive learning case recommendation approach for problem-based e-learning on mathematics teaching for students with mild disabilities, Expert Systems with Applications, 36(3), 5456-5468.
- [3] Bobadilla, J., Serradilla, F., Hernando, A. and Lens, M., (2009). Collaborative filtering adapted to recommender systems of e-learning, Knowledge-Based Systems, 22 (4), 261-265.
- [4] Chang, T.Y. and Chen, Y.T., (2009). Cooperative learning in E-learning: A peer assessment of student-centered using consistent fuzzy preference, Expert Systems with Applications, 36 (4), 8342-8349.
- [5] Baylari, A. and Montazer, Gh.A., (2009). Design a personalized e-learning system based on item response theory and artificial neural network approach, Expert Systems with Applications, 36(4), 8013-8021.
- [6] Junuz, E.,(2009). Preparation of the learning content for semantic e-learning environment, Procedia Social and Behavioral Sciences,1 (1), 824-828.

- [7] Mahdavi, I., Fazlollahtabar, H., Tajdin, A. and Shirazi, B. (2008). Adaptive statistical analysis on higher educational systems, Journal of Applied Sciences, 8 (17), 2998-3004.
- [8] Fazlollahtabar, H. and Sharma, N.K. (2008). E-Learning versus Face-to-Face Learning: An Economic Analysis of Higher Educational Systems in Iran, International Journal of Cyber Society and Education, 1 (1), 49-60.
- [9] Mahdavi, I., Fazlollahtabar, H. and Yousefpoor, N. (2008). Applying mathematical programming and GRA technique to optimize e-learning based educational systems: implementation and teaching skills, Proceedings of the 5th WSEAS/IASME international conference on Engineering education, Heraklion, Greece, 127-131.
- [10] Mahdavi, I., Fazlollahtabar, H. and Yousefpoor, N. (2008). An Integrated Mathematical Programming Approach and Artificial Intelligence Techniques for Optimizing Learning Elements in E-Learning Based Educational Systems, International Journal of Education and Information Technologies, 2 (1), 87-94.
- [11] Fazlollahtabar, H. and Yousefpoor, N. (2009). Cost Optimization in E-learning-Based Education Systems: implementation and learning sequence, E-Learning, 6(2), 198-205.
- [12] Fazlollahtabar, H. and Mahdavi, I. (2009). User/tutor optimal learning path in e-learning using comprehensive neuro-fuzzy approach, Educational Research Review, 4 (2), 142-155.
- [13] Bellman, R.E. and Zadeh, L.A., (1970). Decision-making in a fuzzy environment, Management Science 17, 141–164.
- [14] Okada, S. and Gen, M., (1993). Order relation between intervals and its applications to shortest path problem, in: Proc. 15th Ann. Conf. on Computers and Industrial Engineering 25.
- [15] Kaufmann, A., and Gupta, M.M., (1991). Introduction to Fuzzy Arithmetic: Theory and Applications, Van Nostrand-Reinhold, New York.
- [16] Yamakawa, T., Kusanagi, H., Uchino, E. and Miki, T. (1993). A new Effective Algorithm for Neo Fuzzy Neuron Model. Proceedings of Fifth IFSA World Congress, 1017-1020.
- [17] Lee, W., Hluchyj, M. and Humblet, P. (1993). Rule-Based Call-by-Call Source Routing for Integrated Communication Networks. Proceedings of the IEEE INFOCOM, 987-993.
- [18] Chan, S.C., Hsu, L.S.U. and Loe, K.F. (1993). Fuzzy Neural-Logic Networks, Between Mind and Computer, Fuzzy Science and Engineering, ed. P. Z. Wang and K. F. Loe, World Scientific Pub. Co. Pte. Ltd.
- [19] Buckley, J.J., and Hayashi, Y. (1994). Fuzzy Neural Networks, Fuzzy Sets, Neural Networks and Soft Computing, ed. R. R. Yager and L. A. Zadeh, Van Nostrand Reinhold, NY, 233-249
- [20] Chen, Y. C. and Teng, C. C. (1995). A Model Reference Control Structure Using A Fuzzy Neural Network, Fuzzy Sets and Systems (73), 291–312.
- [21] Chen, Y. C. and Teng, C. C. (1998). Fuzzy Neural Network Systems in Model Reference Control Systems, in Neural Network Systems: Technique and Applications (6), Edited by C. T. Leondes, Academic Press, Inc., 285-313.
- [22] Chen, Y. C. and Teng, C. C. (2001). Fine Tuning of Membership Functions for Fuzzy Neural Systems, Asian Journal of Control 3 (3), 18-25.

- [23] Lee, C. H. and Teng, C. C. (2002). Tuning PID Controller of Unstable Processes: A Fuzzy Neural Network Approach, Fuzzy Sets and Systems 128 (1), 95-106.
- [24] Metropolis, N. and Ulam, S. (1949) 'The Monte Carlo method', Journal of the American Statistical Association, Vol. 44, pp.335–341.
- [25] Sadeghpour Gildeh, B. and Gien, D., (2001). La distance-Dp,q et le cofficient de corrélation entre deux variables aléatoires floues, Actes de LFA'2001, 97-102, Monse-Belgium.

## **Author Introduction**



Hamed Fazlollahtabar is on the faculty of Industrial Engineering at the Mazandaran University of Science and Technology in Iran. He received his MSc in Industrial Engineering from Mazandaran University of Science and Technology. He has awarded a PhD from the Gulf University of

Science and Technology in Kuwait. Currently he is studying PhD of Industrial Engineering at Iran University of Science and Technology, Tehran, Iran. He is on the editorial board of the World Academy of Science Engineering Technology Scientific and a member of the International Institute of Informatics and Systemics. He currently has become a member of Iran Elite Council. His research interests include optimization in manufacturing systems, operation management and information technology management. He has published over 100 papers in journals, books, and conferences.

Tabande Sherafat received his B.Sc. in Information Technology Engineering at the Mazandaran University of Science and Technology, Babol, Iran. She has published his work in Journals and conferences. Her research interest is information technology management.

Alireza Es'haghzadeh received his B.Sc. in Industrial Engineering at the Mazandaran University of Science and Technology, Babol, Iran. He is currently pursuing MSc in Global Production Management at Norwegian University of Science and Technology, Norway. He has published his work in Journals and conferences such as the International Journal of Advanced Manufcturing Technology. His research interest is simulation modeling and applications.